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Three-dimensional drawings in isometric conditions: relation between geometry and kinematics

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Summary. Normal human subjects grasped a 3-D isometric handle with an otherwise unrestrained, pronated hand and exerted forces continuously to draw circles, ellipses and lemniscates (figure-eights) in specified planes in the presence or absence of a 3-D visual force-feedback cursor and a visual template. Under any of these conditions and in all subjects, a significant positive correlation was observed between the instantaneous curvature and angular velocity, and between the instantaneous radius of curvature and tangential velocity; that is, when the force trajectory was most curved, the tangential velocity was lowest. This finding is similar to that obtained by Viviani and Terzuolo (1982) for 2-D drawing arm movements and supports the notion that central constraints give rise to the relation between geometric and kinematic parameters of the trajectory.

Key words: Force – Trajectory – Kinematics – Human

Introduction

A given end-point trajectory of arm movement either in free space or in two-dimensional space can theoretically be produced in a large number of ways. This is known as the problem of the excess of degrees of freedom of the motor system (Bernstein 1967). However, the motor system does not seem to use all degrees of freedom available. Indeed, stable covariations between movement parameters have been documented (Soechting and Lacquaniti 1981; Viviani and Terzuolo 1982; Jeannerod 1984; Flash and Hogan 1985; Soechting et al. 1986) which effectively reduce the number of degrees of freedom of the system. One such covariation relates to geometrical and kinematic properties of the trajectory; namely, it has been shown that the tangential velocity covaries with the radius of curvature in 2-D (Viviani and Terzuolo 1982) as well as 3-D (Soechting and Terzuolo 1986) drawing movements. Lacquaniti et al. (1983) described this covariation as a two-third power law relating the instantaneous angular velocity A(t) and curvature C(t):

$$A(t) = KC(t)^{2/3} \tag{1}$$

where K is a constant. This law can also be written as a one-third power law relating the instantaneous tangential velocity V(t) and radius of curvature $R(t) = 1/C(t)$:

$$V(t) = KR(t)^{1/3} \qquad R < \infty. \tag{2}$$

This relation describes quantitatively the fact that the hand slows down when the trajectory is more curved. In the case of drawing an ellipse, the Eqs. (1) and (2) above imply that the vertical and the horizontal components of the movement are harmonic functions (Viviani and Cenzato 1985). Nevertheless, this law also holds for drawings of other kinds of simple geometrical figures (e.g. the number 8) as well as scribbles (Lacquaniti et al. 1983) in which the vertical and horizontal components of the movement are not harmonic functions. Though systematic departures from the one-third power law have been observed in relation to speed (Wann et al. 1988) or developmental factors (Viviani and Schneider 1991), a power law (with a slightly different exponent) was in good agreement with the data also in these cases.

What is the origin of this covariation between geometrical and kinematic parameters of the movement? Viviani and Terzuolo (1982) suggested that this covariation might emerge from central computational constraints pertaining to the translation of the movement trajectory into the appropriate motor parameters. However, peripheral, biomechanical factors can also be involved. For example, complex interactional forces (e.g. inertial, centripetal and Coriolis forces) are present during multijoint limb movements (Hollerbach and Flash 1982) and it has been shown that the viscoelastic properties of the limb contribute significantly to determine the kinematics of the actual end-point trajectory (Flash 1987; Wann et al. 1988). Thus the biomechanical properties of the arm, which include the oscillatory properties of viscoelastic bodies, may be

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thought to be involved in the relation between curvature and velocity. Finally, the reduction of the speed when negotiating a curve could be programmed explicitly, for it decreases the centrifugal force due to the arm inertia and keeps the arm on-track, which has an apparent adaptive value. In summary, computational and biomechanical constraints, as well as adaptive considerations, may all account for the covariation between geometric and kinematic parameters of the movement. It is then interesting to test this relation in conditions without overt movements of the arm, in which the trajectory is defined in isometric force space and in which the inertia of the arm is not a pertinent parameter for the production of the trajectory: if the covariation between curvature and velocity is still present, then it may indeed originate from central computational constraints. We tested this idea in the present experiments and found that curvature and velocity are still correlated under isometric conditions. Preliminary results were reported (Lurito et al. 1990).

Methods

Subjects

Twenty-six healthy, unpracticed human subjects (6 females and 20 males) participated in this experiment. Subjects performed the task with their preferred hand; all subjects but one were right-handed.

Apparatus

The experimental apparatus consisted of a 3-D isometric manipulandum and a color video monitor on which stereographic figures and a force-feedback cursor could be displayed. The devices and methods for generating the stereograms were described previously (Massey et al. 1988, 1991). Briefly, the manipulandum was a vertical T-shaped metal handle which the subjects grasped with the hand pronated. The vertical rod was 18 cm long and was mounted perpendicular to the plane of three load cells. The force exerted in each of the three axes (X, Y and Z) was digitized and sampled every 10 ms. The maximum force range of the load cells was ± 2000 gm-force with a measured accuracy of ± 4 gm-force. A force-feedback cursor could be displayed in correspondence to the forces exerted on the manipulandum; 100 gm-force produced a displacement of the force-feedback cursor of 1 cm. Figures and cursor, when present, were displayed on the color monitor as anaglyphic stereograms viewed by the subjects through appropriate filters (Massey et al. 1988).

Subjects sat comfortably with the chin in a chin rest. The chair height, chin rest and display position were individually adjustable. The manipulandum was placed in front of the subjects at the midsagittal plane and approximately at the elbow level with the upper-arm along the body.

Behavioral task

Subjects grasped the manipulandum with the hand pronated and the fore- and the upper-arm unrestrained. They were asked to exert force to draw continuously, at a self-chosen rhythm, ellipses or lemniscates (figure-eights) in a frontal (XZ), horizontal (XY) or sagittal (YZ) plane. In some cases a figure and the force-feedback cursor were presented on the screen; however, the subjects were instructed not to attempt to trace exactly the form and size of the figure but, instead, to make continuous, rhythmic motions in duplicating the type of the figure. In other cases no figure or cursor were presented, but the subjects were instructed verbally concerning the type of the figure and the plane in which to draw it. Not all subjects produced every figure or performed under both visual conditions. Trials usually lasted ten seconds. Data were recorded during the last five seconds, after the subject reached a stable rhythm.

Data analysis

The force data were smoothed using a digital filter with a low-pass cut-off of 15 Hz. Instantaneous force velocity and acceleration were obtained by digital differentiation using a five point polynomial approximation. The instantaneous angular velocity A and curvature C were computed and the generalized power law was tested:

$$A(t)=KC'(t)^{\beta} \quad (3)$$

where K is a constant, $C'(t)=1/R'(t)$, $R'(t)=R(t)/(1+\alpha)$, and $\alpha=10^{-4}$ (see below). Equation 3 is linear in a log scale:

$$\log_e(A)=\log_e(K)+\beta\log_e(C') \quad (4)$$

where $\log_e$ indicates the natural logarithm. (In the remaining of the paper "log" is used instead of "$\log_e$".) We used $R'(t)$ instead of $R(t)$ in order to deal with data points in which the curvature tended to zero; the constant α was chosen to be arbitrarily small.

The instantaneous tangential velocity and radius of curvature were computed and the corresponding power-law was tested:

$$V(t)=KR'(t)^{\delta} \quad (5)$$

where K is a constant. Equation 5 is linear in a log scale:

$$\log(V)=\log(K)+\delta\log(R') \quad (6)$$

The strength of the relations (Eqs. 4 and 6) was quantified by calculating the correlation coefficient *r*.

Results

General

Examination of the data revealed that the subjects exerted 3-D forces which generally reproduced the instructed figures and were smooth and obviously periodic with average frequencies ranging from 0.14 to 2.86 Hz, and with an average of 0.87 Hz for the circles and ellipses and of 0.90 Hz for each loop of the lemniscates. The average time taken to complete a relatively simple figure (circle or ellipse) was less than that to complete the lemniscates, 1.55 s and 2.57 s respectively. Fourier analyses of data samples indicated that, in general, the second harmonic was very small and that higher harmonics were not present. Average force velocities ranged from 800 to 14000 gm-force/s and average perimeters from 1632 to 23040 gm-force.

An example of force trajectories is shown in Fig. 1. In the left column, the force trajectories are shown in perspective view. The projections of the trajectories onto the three cardinal planes are shown in the right panel. The data illustrated were obtained from the same subject who was instructed to draw an ellipse in the sagittal (YZ) plane (top row), a lemniscate in the frontal (XZ) plane (middle row), and a lemniscate in the sagittal (YZ) plane (bottom row). The instructed shapes are shown by solid lines whereas the data points, collected every 10 ms, are shown as dots. The arrows indicate the direction of drawing. It

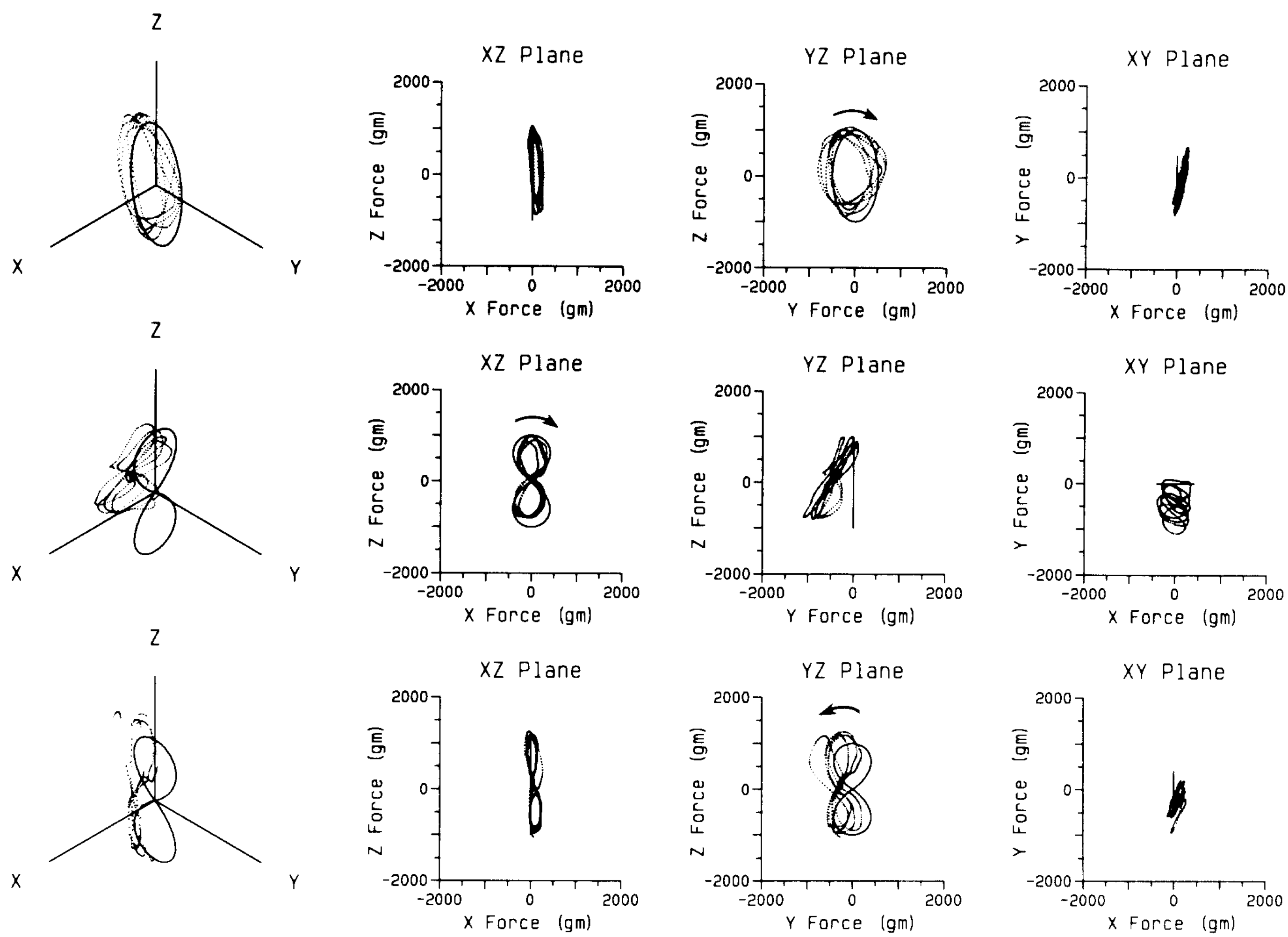

Fig. 1. Force trajectories performed by one subject are shown as spatial plots. See text for explanation

can be seen that instructed shapes were generally reproduced in the planes specified but distortions were also present. In this paper we concentrate on the relations between geometric and kinematic parameters of the trajectories; the spatial aspects of the figural trajectories will be considered elsewhere.

Relation between geometric and kinematic parameters

In the first row of the Fig. 2 a different set of data is illustrated, and in the second row, the XYZ components of force for these data are plotted against time. The relation between geometric and kinematic parameters for these data are presented in the Fig. 3. In the left panel of the first row, the angular velocity (solid line) and curvature (dashed line) are plotted against time; it can be seen that they clearly covary. This covariation is documented in the right panel, which shows a scatter plot between these two variables. In this case the regression equation was:

$$\log(\mathrm{A}) = -0.196 + 0.707 \log(\mathrm{C}') \qquad (7)$$

$$(r = 0.863)$$

or, equivalently,

$$\mathrm{A}(t) = 0.822\mathrm{C}'(t)^{0.707}. \qquad (8)$$

In the left panel of the second row, the log-transformed tangential velocity (solid line, labeled V) and radius of curvature (dashed line, labeled R) are plotted against time. In the scatter plot on the right panel, in which these two variables are plotted against each other, the regression equation was:

$$\log(\mathrm{V}) = -0.412 + 0.325 \log(\mathrm{R}') \qquad (9)$$

$$(r = 0.815)$$

or, equivalently,

$$\mathrm{V}(t) = 0.663\mathrm{R}'(t)^{0.325}. \qquad (10)$$

The covariation between the angular velocity and curvature, and between the tangential velocity and the radius of curvature was observed in all subjects and under all conditions. The frequency distribution of the exponent for the first relation above and for the whole set of data ($N = 202$ cases) is shown in Fig. 4. The mean ($\pm$SD) of the exponent was 0.763 ± 0.094 for the first relation, and 0.326 ± 0.093 for the second one.

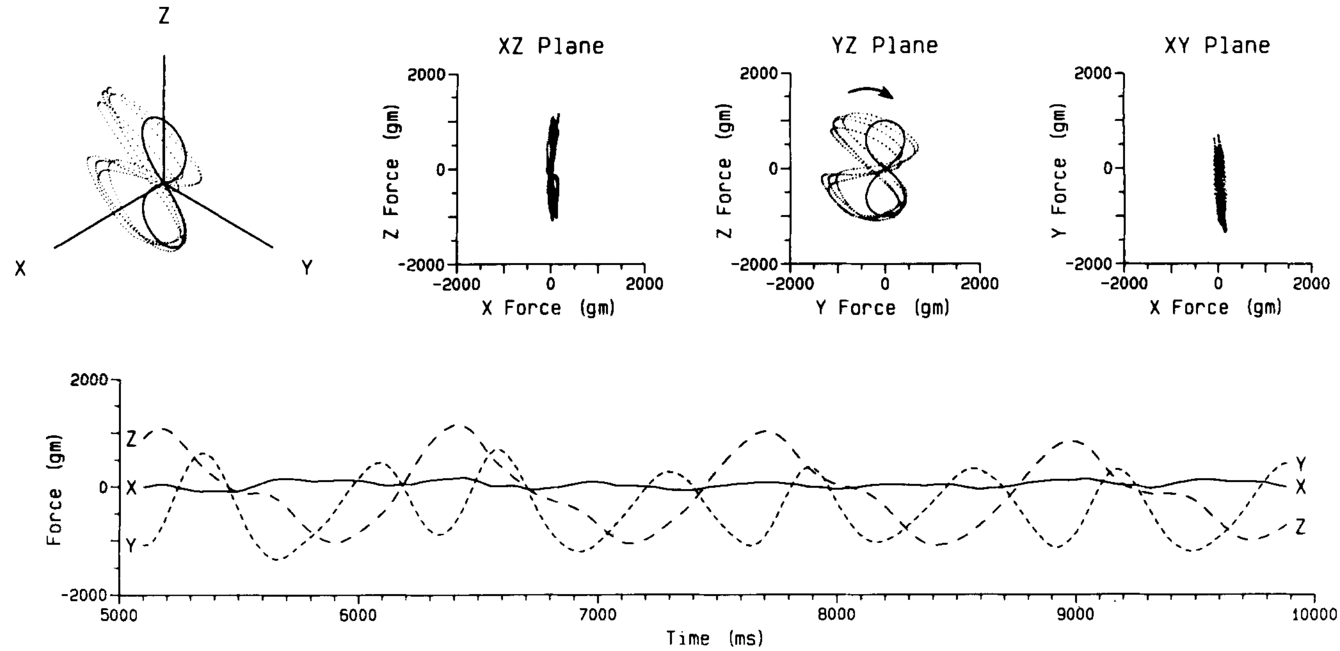

Fig. 2. Force trajectory performed by one subject. *First row*: spatial plots (as in Fig. 1). *Second row*: the XYZ components of force are plotted against time

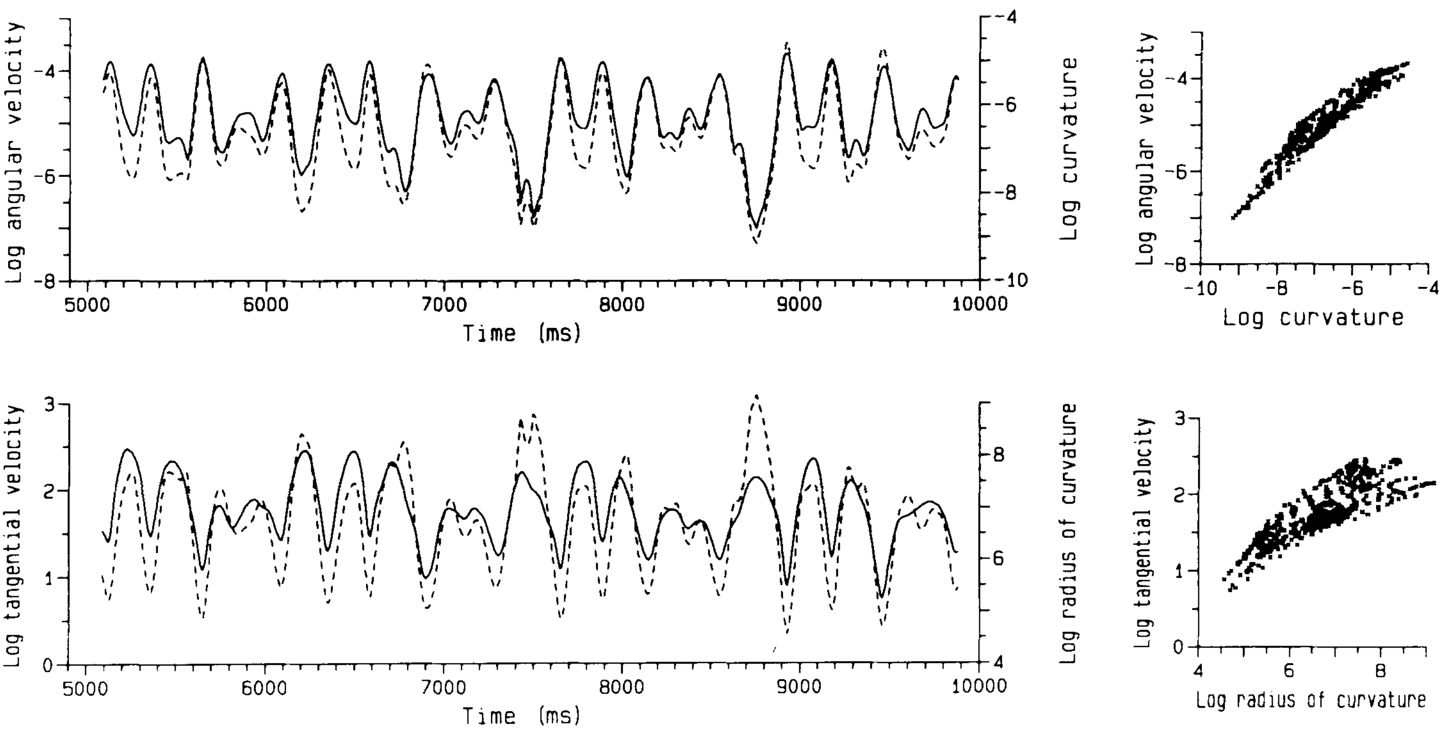

Fig. 3. Relation between geometric and kinematic parameters for the data presented in Fig. 2. *First row, left panel*: the log-transformed angular velocity (solid line) and curvature (interrupted line) are plotted against time. *First row, right panel*: these two variables are plotted against each other. *Second row, left panel*: the log-transformed tangential velocity (solid line) and radius of curvature (interrupted line) are plotted against time. *Second row, right panel*: these two variables are plotted against each other

It can be seen in the scatter plots in Fig. 3 that the relation of A vs. C (first row) is more tight than that of V vs. R (second row); indeed, the cluster is tighter and the correlation coefficient higher in the first case. This was consistently observed in 200/202 (99%) cases.

Discussion

The major finding of this study is that the covariation between tangential velocity and radius of curvature observed by Viviani and Terzuolo (1982) for drawing move-

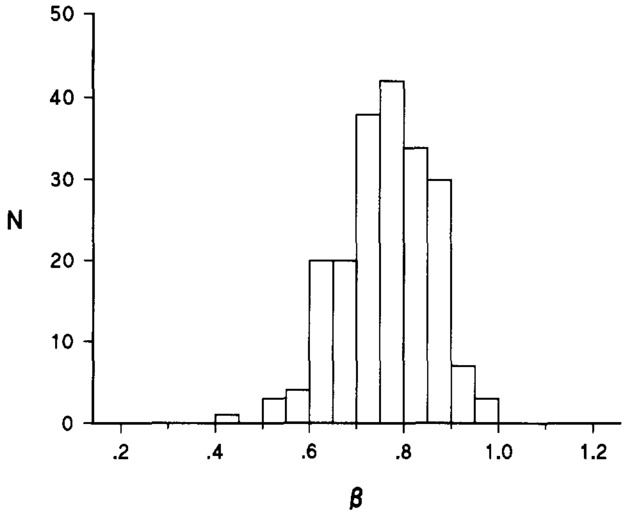

Fig. 4. Frequency distribution of exponents obtained for the relation between the angular velocity and the curvature (see Eq. 3) and for the whole set of data ($N = 202$)

ments of the arm is also present under isometric conditions, that is when trajectories were in force space, in the absence of movement of inertial loads. Moreover, the average value (0.326) of the exponent in the power law (Eq. 3) was close to 1/3 (see Eq. 2) and therefore similar to the value frequently obtained in arm movement experiments. This finding suggests that the covariation between geometric and kinematic parameters is of central origin and cannot be attributed to peripheral biomechanical factors arising from moving the inertial load of the arm; such factors may contribute but they are not of primary importance for the existing relation.

It is interesting that the relation between angular velocity and curvature was stronger than the one between tangential velocity and radius of curvature. This suggests that the angular velocity may be a crucial parameter for the generation of the motor command.

Neural constraints for changing direction

Two properties of the relation between geometric and kinematic parameters of the trajectory are captured in the Eqs. (1) and (2), and in their general expression (3) and (5). One is related to a global property of the movement (viz the tempo) and depends on the gain factor K. The other one is related to the local differential properties and is expressed by the power law itself and its exponent. As we suggest below, both of these properties can be tentatively related to neural constraints.

Global properties. Making a curve in a trajectory involves changing direction; and changing or specifying direction is a time-consuming process. For example, precuing for movement direction results in substantial savings in reaction time (Rosenbaum 1980). Moreover, increases in reaction time have been observed in tasks in which the direction of the movement relatively to a visual target (Georgopoulos and Massey 1987) or the orientation of a visual image (Shepard and Cooper 1982) have to be transformed. These tasks seem to involve a mental rotation (i.e. change of direction). It is interesting to note that the average rate of rotation for movement and letter-rotation tasks are correlated across subjects, which suggests that these tasks share common processing constraints (Pellizzer et al. 1991). The neural correlates of the movement-rotation task were studied in monkeys by recording the activity of single cells in the motor cortex while the animals planned movements at 90° counterclockwise (CCW) from a stimulus direction (Georgopoulos et al. 1989). The results were analyzed using the neuronal population vector (Georgopoulos et al. 1983) which is a measure of the directional tendency of the neuronal ensemble in both movement (Georgopoulos et al. 1983, 1984, 1986, 1988; Kalaska et al. 1989; Caminiti et al. 1990) and isometric (Taira et al. 1991) tasks. In pointing movements, the population vector, calculated every 20 ms during the RT, points in the direction of the upcoming movement (Georgopoulos et al. 1984, 1988). During the RT of the 90°CCW task, the population vector pointed initially in the direction of the stimulus and then rotated in a linear fashion by approximately 90°CCW to point in the direction of the movement (Georgopoulos et al. 1989). Interestingly, the slope of the rotation of the population vector was in the same range as the one observed in the RT vs. angle relation in human subjects performing the movement-rotation task; in recent experiments in two monkeys the average slope of the rotation of the population vector was $418 \pm 175°/s$ (Lurito et al. 1991). In summary, all of these results indicate that central processes involving change in movement direction operate at an average angular velocity (i.e. rotation rate) of approximately 400°/s. Now, it is remarkable that in the present experiments the overall average cycle frequency was 0.9 Hz, corresponding to an average angular velocity of 400°/s. It is therefore possible that the average angular velocity when performing a curved trajectory simply reflect a neural constraint, that is the average rate of rotation of the neuronal population vector. A corollary of this idea is the tendency for subjects to complete curved figures of different size at an approximately equal rate (i.e. isochrony principle). Indeed, it has been shown that in drawing circles of different sizes, the average tangential velocity increases almost proportionally to the perimeter so that the time taken to complete a whole circle is approximately constant (Viviani and McCollum 1983). The rate to complete a closed trajectory is approximately 1 Hz (Wann et al. 1988; Viviani and Schneider 1991) which is close to the average rotation rate of the population vector.

Local properties. The instantaneous direction and length of the neuronal population vector seems to reflect, with a time delay, the instantaneous tangential velocity of the movement. Indeed, when a time series of consecutive neuronal population vectors are strung together the "neural" trajectory thus obtained is close to the actual reaching movement trajectory (Georgopoulos et al. 1988). Moreover, it has been shown by Schwartz and Anderson (1989) that when monkeys trace sinusoidal trajectories with the hand, the change in time of the neuronal population vector predicts well the corresponding ensuing sinusoidal trajectory. The "neural" trajectory precedes the actual movement trajectory with a delay of approximately 150 ms (Georgopoulos et al. 1988; Schwartz 1988). These findings indicate that neural constraints probably underlie the observed local properties of the relation between geometric and kinematic parameters of drawing movements. However, the neural basis which would explain why this relation follows a power law of relatively constant exponent remains to be explored.

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